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GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

# STATUS REPORT # 2

on

# THEORY OF SPACE FLIGHT AND ADAPTIVE GUIDANCE

Coordinated By

W. E. Miner.

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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ABSTRACT 20206

An organized effort is presently being conducted by Future Projects Branch, involving a group of both in-house and contractor personnel, for the purpose of advancing the theory of space flight and adaptive guidance, and developing and improving the techniques of their application to present and potential Saturn missions. Status Report #1 introduced the scientific disciplines involved and broadly outlined the overall working philosophies of the group. The present report summarizes and comments on the results obtained by the group to date in the involved scientific disciplines of Celestial Mechanics, Calculus of Variations, Large Computer Exploitation, and their application.

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FUTURE PROJECTS BRANCH
AEROBALLISTICS DIVISION

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# STATUS REPORT #2
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SUMMARY

An organized effort is presently being conducted by Future Projects Branch, involving a group of both in-house and contractor personnel, for the purpose of advancing the theory of space flight and adaptive guidance, and developing and improving the techniques of their application to present and potential Saturn missions. Status Report #1 introduced the scientific disciplines involved and broadly outlined the overall working philosophies of the group. The present report summarizes and comments on the results obtained by the group to date in the involved scientific disciplines of Celestial Mechanics, Calculus of Variations, Large Computer Exploitation, and their application.

# INTRODUCTION

by
W. E. Miner

The following material presents the efforts of the Future Projects Branch of the Aeroballistics Division, its contractors, and associates during the period from March 1962 through November 1962.

Here we do not propose to give the background material for the studies. Status Report #1 should be read for this background. The paragraphs which follow in this introduction summarize the work in the various fields of study.

The first paper by Mirt Davidson on "Celestial Mechanics" points up two things. First, Dr. R. F. Arenstorf of the Computation Division has made significant contribution to the field in his note "Periodic Solutions of the Restricted Three Body Problem Representing Analytic Continuation of Keplerian Elliptical Motion." Second, progress has been made both in-house and out-of-house in obtaining insight into the problems of earth-moon trajectories. In-house, Mirt Davidson displays a simple geometrical property when considered as a function of the special initial values. Republic Aviation is continuing work on the Hamilton-Jacobi approach using the Euler fixed center problem as a base.

Two approaches have been discarded as having no merit. In the engineering sense of developing cutoff equations, we are still far away from solutions and will start work on some empirical procedure based on special perturbation procedures such as is being developed by Dr. Hans Sperling.

The paper on "Calculus of Variations" presents three main areas of work. The first is that of studies into the transversality conditions at staging points. The second is that of the development of means to check the Jacobi conditions (uniqueness of solution) needed for a sufficiency check on the trajectory. The latter work was done by Dr. Robert Hunt while he was with us. The last section discusses some work started in developing procedures for reentry studies. This process is not in the state of development where it could be called a true application. It may, in general, be stated that the studies in calculus of variations are progressing very well.

The work reported on by Nolan Braud in "Large Computer Exploitation" shows three areas of accomplishment. The first is that of the function

differential generator. It is hoped that this tool will be refined in order to increase its usefulness. At present we have a tape with the operation defined and are using it to obtain differentials. The differentials generated are to be used in-house for numerical methods of defining steering equations. The second item of importance is the developments toward the use of linear programming in determining the steering function from empirical data. The potential here is that of better control of error. The last major accomplishment is that of developing better means for least squares fits by use of orthonormal polynomials. This approach allows greater control of the accuracy of approximation and should alleviate the requirement of using residual procedures to improve accuracy. The condition for existence of multivariable least squares approximating polynomials developed by Northeast Louisiana State College indicates that no check needs to be made for this condition.

The applications paper presents some of the major areas of work in the three project sections of our branch. This work is detailed in reports of the branch and only a review is given here. These major study areas are:

1. Studies for Manned Lunar Flight through Lunar Orbital Rendezvous,

2. Lunar Logistics System Studies, and

3. Adaptive Guidance Applications.

Some of the work of Boeing Huntsville is reported. This work is on coasting periods during the ascent phase.

In conclusion, it may be noted that the weakest area of endeavor (from the standpoint of results useful in the applications) is that of celestial mechanics. Notable progress is being made in calculus of variations and large computer exploitation.

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# CELESTIAL MECHANICS

By

M. C. Davidson

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# CELESTIAL MECHANICS

By

M. C. Davidson

## SECTION I. INTRODUCTION

This section gives a brief survey of the progress made, both in-house and out-of-house, in the field of celestial mechanics since the publication of the preceding status report (Reference 1). During this time significant advances have been made, especially in the study of periodic motion. Here at Marshall Dr. Arenstorf has given an existence proof for periodic orbits of the second kind, answering a significant question which dates back to the nineteenth century. As a result of a special representation of the solutions of the restricted three body problem, we are able to infer a geometrical property. These results and others are to be discussed in the following sections.

## SECTION II. PERIODIC MOTION IN THE RESTRICTED THREE BODY PROBLEM

### A. INTRODUCTORY COMMENTS

Let the differential equations for the restricted three body problem in the usual rotating cartesian coordinate system be written as

$$\dot{x}_k = f_k\ (x_1, x_2, x_3, x_4, \mu)\ , \quad (k = 1 \ldots 4), \tag{1}$$

where $x_1$ and $x_2$ are the position coordinates, $x_3$ and $x_4$ are the corresponding momenta, and $\mu$ is the mass ratio, (Reference 4). Since the system (1) is autonomous the conditions for a solution, $x_k(t, \xi, \mu)$, to be periodic with period $\tau$ may be written as

$$F_k\ (\tau, \xi, \mu) = x_k\ (\tau, \xi, \mu) - \xi_k = 0, \quad (k = 1 \ldots 4), \tag{2}$$

where $\xi$ represents the four initial values

$$\xi_k = x_k\ (t = 0)\ , \quad (k = 1 \ldots 4).$$

In certain cases a symmetry property in the restricted problem allows us to reformulate condition (2). It is known that if an orbit cuts the $x_1$ axis orthogonally, then this orbit is symmetrical with respect to the $x_1$ axis. If we take the initial values to be at such a crossing, condition (2) may be written as

$$x_2 \left(\frac{T}{2}, \xi_1, \xi_4, \mu\right) = 0 \qquad (3)$$

$$x_4 \left(\frac{T}{2}, \xi_1, \xi_4, \mu\right) = 0$$

where $\xi_2$ and $\xi_3$ are taken to be zero.

Let us now discuss a theorem that allows us under certain conditions to invert implicit equation such as (2) and (3).

In our case the implicit function theorem may be stated as follows: let the functions

$$F_k \, (y_1, \quad , y_{m+2}) \quad , \quad (k = 1 \ldots m),$$

be holomorphic in all m+2 variables at the point

$$P^* = \left\{ \left| y_1^*, \ \ldots, \ y_{m+2}^* \right\} \right| \quad .$$

Further, suppose the $P^*$ is a particular solution to the system

$$F_k = 0 \quad , \quad (k = 1 \ldots m).$$

Then, the functions

$$y_k = y_k \, (y_{m+1}, y_{m+2}) \ , \quad (k = 1 \ldots m) \ ,$$

exist as holomorphic functions of the two complex variables $y_{m+1}$ and $y_{m+2}$ in a neighborhood of the point

$$Q^* = y^*_{m+1}, \ y^*_{m+2} \quad ,$$

so long as the Jacobian,

$$D^* = \left| \frac{\partial(F_1 \ldots F_m)}{\partial(y_1 \ldots y_m)} \right| \neq 0$$

where D* is evaluated at the point P*.

In applying this theorem to either (2) or (3), the holomorphic conditions are referred to the Cauchy-Poincaré existence theorem (Reference 6).

The establishing of a particular solution is possible by use of the Kepler problem, $\mu = 0$. In this case, $\mu = 0$, and equations (1) are the differential equations for Kepler motion referred to a rotating coordinate system rotating with angular speed one. There are two possibilities; first, if the Kepler motion is circular in an inertial frame, this motion referred to the rotating system is also periodic and symmetric and, hence, is a solution of (2) and of (3); second, if the inertial motion is a proper ellipse, its period must be commensurable with $2\pi$ to be a solution of (2) and of (3). From these solutions, provided that one of the possible D*'s does not vanish, we may perform the necessary, analytic continuation producing periodic solutions for small $\mu > 0$.

If the analytic continuation is started from a circular solution for $\mu = 0$, the periodic solution thus obtained for $\mu > 0$ is said to be of the first kind. Similarly, for elliptic solutions we obtain periodic solutions of the second kind.

## B. PERIODIC SOLUTIONS OF THE FIRST KIND

It is a classical result that there exist periodic orbits of the first kind for small $\mu > 0$ so long as the inertial period of the original circular solution is not the reciprocal of a non-zero integer. This proof was given by Poincaré in 1892 and produces a one parameter family of such orbits. Poincaré made the continuation in an isoperiodic manner. Later continuation was carried out in a isoenergetic manner; however, this produced the same family.

Siegel has shown, (Reference 6), that isoperiodic solutions of the first kind exist about both masses for sufficiently large values of the original circular period and all $\mu$ in [0, 1]. The Fourier series thus obtained allows one to construct the initial values of such orbits.

At present there is no method of constructing these orbits for all $\mu$ in [0, 1] and all values of the circular period not equal to the reciprocal of the non-zero integer.

## C. PERIODIC SOLUTIONS OF THE SECOND KIND

The historical background of this problem is large and is given in Reference 3. It suffices to say that more than one mathematician of renown, including Poincaré and Birkhoff, have published incorrect proofs of their existence.

Here at Marshall Dr. Arenstorf, by the use of suitable variables, condition (3), and the implicit function theorem has given a valid proof (Reference 3). He has shown that for a given $k/m$, $m > 0$, where k and m are non-zero integers, and the inertial period of the elliptic motion equals $k/m$, that there exists periodic solutions of the second kind with finitely many exceptions of the eccentricity $\epsilon$ for $0 < \epsilon < 1$.

# SECTION III. PERTURBATION TECHNIQUES

The formal perturbation method of Hamilton-Jacobi, applied to the restricted problem is being pursued by Republic Aviation. This process is quite lengthy, but does offer some hope of gaining information about the problem.

The in-house effort based on canonical initial values has been at least temporarily terminated. The use of the functions coming from this method to represent solutions to the initial value problem offered no significant advantage over simpler fitting techniques; however, the following result has its origin in these studies.

Let the solutions of (1) be written as

$$z(t, \xi) = \begin{pmatrix} x_1(t,\xi) \\ x_2(t,\xi) \\ x_3(t,\xi) \\ x_4(t,\xi) \end{pmatrix}, \qquad \text{(P1)}$$

with initial values

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{pmatrix}$$

Suppose there exists a function $w(t, \eta)$ and a $t^* > 0$ such that

$$z\ (t,\ \xi) = w\ (t,\ R(t),\ \xi)\ ,\quad 0 \leqq t \leqq t^*, \qquad \text{(P2)}$$

where $w(t, \eta)$ satisfies an autonomous differential equation, say

$$\dot{w} = g(w)$$

for arbitrary initial values $\eta$. Further, the four by four matrix $R(t)$ is to satisfy

$$R(t_1)\ R(t_2) = R(t_1 + t_2)$$

for all $t_1$ and $t_2$ in the interval $[-t^*, t^*]$. Then the following is true (Reference 5).

If $z(t, \xi)$ be the value of z at time t with initial values $\xi$, then the values of $z(t, R(\rho)\ \xi)$ at time t with initial values $R(\rho)\xi$ is $R(\rho)z(t, \xi)$ for all $\rho$ and t such that

$$|\rho| + |t| \leqq t^*.$$

The perturbation method of canonical initial values (References 2 and 4) gives such a functional relationship where $w(t, \eta)$ represents the solution of Euler's problem of two fixed centers. Here the matrix $R(\rho)$ is

$$R(\rho) = \begin{pmatrix} \cos\rho & \sin\rho & 0 & 0 \\ -\sin\rho & \cos\rho & 0 & 0 \\ 0 & 0 & \cos\rho & \sin\rho \\ 0 & 0 & -\sin\rho & \cos\rho \end{pmatrix},$$

which corresponds to rotating the initial values through an angle $\rho$. For visualization, let us call in the restricted problem the mass $1-\mu$, earth and the mass $\mu$, moon ($\mu$ small). Now assume we have an earth to moon trajectory starting with initial values, say $z_o$. Then the one parameter family of trajectories defined by the one parameter set of initial values $R(\rho)\, z_o$ would spread out accordingly as

$$z\,(t,\, R(\rho)\, z_o) = R(\rho)\, z\,(t,\, z_o).$$

On the other hand, suppose we have a moon to earth trajectory starting with initial values near the moon, say $z_o$. Then as before the one parameter set of trajectories is constructed; however, these would have a focusing effect at the earth. Figures 1 and 2 of Reference 5 give such trajectories in the restricted problem.

The expansion of the solutions of (1) with respect to the parameter of mean motion (Reference 1) by the University of Kentucky has been terminated. The difficulty here is that only the first term in the series may be integrated in closed form.

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5. AIN 33-62, "A Local Geometrical Property of Orbits in the Restricted Three Body Problem," dated September 27, 1962, by Mirt C. Davidson.

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# THE CALCULUS OF VARIATIONS

By

Robert Silber and Hugo Ingram

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# THE CALCULUS OF VARIATIONS

By

Robert Silber and Hugo Ingram

## SECTION I. INTRODUCTION

Most of what is needed as introduction to what is to follow can be found in the first Status Report in the section on calculus of variations and the applications of the theory. However, we shall briefly restate certain of the results presented there which will serve to put a proper perspective behind what is to follow.

Recall that the first necessary condition of the classical theory is the so-called Euler-Lagrange equation. The result of the application of this equation is a set of second order differential equations which must be satisfied along any trajectory which minimizes the considered integral.

There are a few things to be said concerning the first necessary condition. First, it should be emphasized that this is not the only known necessary condition. In a certain vague sense, it is the "largest" in that it in many cases restricts the field of candidates to those curves which are solutions to certain differential equations. Accordingly, if a general solution is available to these equations, the problem is reduced to the appropriate selection of the arbitrary constants in this general solution.

However, it is not a priori known that there is a solution to the minimization; i.e., there is no guarantee that some particular solution to the aforementioned differential equations actually minimizes the considered integral. On the other hand, given that there is a solution, it is not evident that there is only one.

The other necessary conditions to the classical theory are concerned with these questions. In addition, they are sometimes helpful in the appropriate determination of the arbitrary constants previously mentioned.

## SECTION II. THE TRANSVERSALITY CONDITION

Recall that the basic problems of the classical theory are built around the minimization (or maximization) of an integral

$$J = \int_{t_0}^{t_1} f(t, x, \dot{x})dt$$

by the appropriate selection of the function

$$x(t) = x_1(t), x_2(t), \ldots, x_n(t)$$

from a certain defined class of "admissible" functions.

The student of the calculus of variations is frequently asked for convenience to think of the function $x(t)$ as describing a curve in $(t, x)$ space and the integral $J$ is thought of as some property of the admissible curves such as their lengths. Such an interpretation will be helpful in discussing the application of the transversality condition.

One of the first problems considered in the calculus of variations is the minimization of the integral $J$ relative to curves $x(t)$ which have fixed end points. Such is the case when finding the curve of minimum length between two fixed points. In this case, one of the properties of the admissible curves is that they must connect the two points under consideration. Of course, it is not difficult to generalize this problem to one of variable end points by attempting to find the curve of minimum length connecting two surfaces. Also, it is not difficult to see that the solution of the problem is still a straight line; it is but necessary to find the appropriate point on each surface to which the end points of the straight line are to be connected.

It is illuminating to realize that for this problem the Euler-Lagrange equations dictate that the solution to the problem shall be a straight line. The general solution to the differential equations resulting from application of the Euler-Lagrange equations is the family of all straight lines. For the fixed end-point problem, we simply determine

the unique straight line defined by the fixed end-points. In the variable end-point problem, the task is reduced to the selection of the shortest line segment connecting a point of one surface to a point of another.

In general, the Euler-Lagrange equations are applicable in the variable end-point problem just as in the fixed end-point problem and yield the same differential equation in both problems.

In the trajectory analysis with which we are concerned, the fixed end-point problem would correspond to the stipulation of all position and velocity coordinates at each end of a given stage. As pointed out in the first status report, this generally results in a two-point boundary value problem, and under normal conditions can be expected to determine a unique solution to the governing differential equations, just as in the case of two points determining a unique line segment.

However, this fixed end-point problem generally does not correspond to the physical problem with which we are confronted. Rather, our problem is generally one of variable end-points. For example, as explained in Status Report # 1, thrust termination for the uppermost stage is defined by a set of mission criteria which state necessary and sufficient conditions for eventual mission fulfillment. There is no reason to expect that there will generally exist exactly one set of state variables at thrust termination satisfying these criteria. (Consider, for example, the mission of obtaining a circular orbit.)

We consider, therefore, that the mission criteria generally define a surface (in phase space) on which the end-point must lie. The number of degrees of freedom of this surface will, of course, depend on the mission statement, i.e., on the particular set of mission criteria.

The transversality condition is a condition applicable in the variable end-point problem. Generally, one obtains one condition for each degree of freedom on the terminal surfaces. In effect, then, the transversality condition will furnish those conditions for selecting the best point of the terminal surface. In this sense, it is a device for determining certain of the arbitrary constants of integration mentioned previously, thereby reducing the problem once again to a two-point boundary value problem.

For several stages, the transversality condition is applicable at the points of junction of successive stages. This is very important from the standpoint of reducing the number of independent parameters under consideration. Dr. Boyce of Vanderbilt University is investigating the possibility of inferring certain general relationships between the undetermined multipliers across stage junctions by employing the transversality conditions. The problem is that, while the transversality conditions do generally define the behavior of the multipliers at such points, the definitive conditions are implicit and further, are dependent on quantities defined by the solution trajectory. Therefore, no direct calculation is at present feasible, and it is necessary to increase the order of the isolations associated with the numerical determination of the solution of the two-point boundary value. Consequently, the only advantage presently offered by the transversality condition (except in certain special cases) is to convert the optimization of the additional parameters encountered in each new stage to an isolation for their proper (optimized) values.

## SECTION III. THE JACOBI CONDITION

Another necessary condition of the theory is the so-called Jacobi condition. This condition is related to the existence and uniqueness of a minimizing trajectory. The application of the condition results in a certain accessory minimum problem which is an associated problem to which the Euler-Lagrange equations are again applicable. However, for this minimization, the application of the Euler-Lagrange equations results in differential equations of a simple type. The equations are in fact linear and homogeneous. That their solutions can be numerically investigated is pointed out by Dr. R. W. Hunt in his report "Utilization of the Accessory Minimum Problem in Trajectory Analysis," (MTP-AERO-62-74). In effect, for a given supposed minimizing trajectory determined by solving the two-point boundary value problem resulting from the Euler-Lagrange equations, the Jacobi condition can be numerically checked at each time point on the trajectory. A program for doing this has been developed by Dr. Hunt and is presently in use.

One of the reasons for the importance of the Jacobi conditions (in addition to being a necessary condition) is that, in combination with certain other conditions of the theory, a condition for sufficiency can be theoretically

determined. Therefore, the check for the Jacobi condition actually provides a step forward in being able to make absolutely certain statements as to the optimality of a given trajectory. The remaining conditions to be met for sufficiency are the Legendre (or Weierstrass) condition and Hilbert's non-singularity condition. It is felt that these checks can be eventually carried out with no principal difficulties.

## SECTION IV. AN APPLICATION OF THE PONTRYAGIN PRINCIPLE TO REENTRY TRAJECTORY OPTIMIZATION

### A. INTRODUCTORY REMARKS

The reentry phase of any type of mission which is designed to return a man to the earth's surface will here be assumed to start at the altitude above the earth's surface at which the earth's atmosphere begins to have a measurable effect upon the flight path of the manned vehicle. The reentry phase is assumed to have ended when parachutes or some other type of landing device can be safely employed.

Some important properties of the reentry flight are the deceleration history and the range covered. These quantities depend most strongly on the reentry conditions and the attitude history of the vehicle. It is desired to eventually be able to determine the extent of variations in reentry conditions that can be tolerated when optimum attitude histories are used to obtain satisfactory decelerations and range coverage. These "reentry windows" are needed to determine guidance in previous phases of the flight profile.

This report covers some first steps in developing decks to do this. The Pontryagin Maximum Principle is applied in computing the optimum reentry trajectories, and some information on the numerical behavior of the Pontryagin H function is obtained. The integral of total drag squared that is extremized here is only one of many possible functions that may be studied. It is not yet known what function is most desirable to be extremized physically.

## B. BRIEF DESCRIPTION OF THE PONTRYAGIN PRINCIPLE

Consider the following system of equations:

$$\dot{x}_i = f_i(x_1, \ldots, x_n; u_1, \ldots, u_r; t) \qquad (i = 1, \ldots, n)$$

where $(u_1, \ldots, u_r)$ are the control variables and $(x_1, \ldots, x_n)$ are the state variables. The quantity

$$S = \sum_{i=1}^{n} c_i \, x_i$$

is to be extremized at some time $(t_f)$ after the initial time $(t_o)$. At $t_o$ the values for $x_1, \ldots, x_n$ are assumed to be known. The $c_i$'s are arbitrary constants and the values which they are assigned determine what combinations of the $x_i$'s are extremized at $t_f$.

In order to specify further the manner in which

$$S = \sum_{i=1}^{n} c_i \, x_i \, (t_f)$$

is to be extremized, the adjoint system of equations is defined:

$$\dot{\lambda}_i = - \sum_{j=1}^{n} \lambda_j \frac{\partial}{\partial x_i} \left[ f_j(x_1, \ldots, x_n; u_1, \ldots, u_r; t) \right] (i=1, \ldots, n) \; .$$

Introducing the function $H = \lambda_i f_i \; (i = 1, \ldots, n)$ allows the original system and the adjoint system of equations to be written in the following form:

$$\dot{x}_i = \frac{\partial H}{\partial \lambda_i} \quad \text{and} \quad \dot{\lambda}_i = - \frac{\partial H}{\partial x_i} \qquad (i = 1, \ldots, n) \; .$$

Now the Pontryagin Principle can be stated in the following form. If S is to be a minimum at $t_f$, it is necessary that the control variables ($u_1$, ..., $u_r$) be chosen at $t_o$ and every time thereafter in such a way that H at each such fixed time is a maximum with respect to these control variables. If it is desired that S be a maximum at $t_f$, then H must be a minimum with respect to $u_1$, ..., $u_r$ at every $t_o \leq t \leq t_f$. Associated with the Pontryagin Principle are relations which determine the initial values for the $\lambda_i$'s in the following three cases.

Case 1 If none of the $x_1$, ..., $x_n$ are desired to have an assigned value at a fixed $t_f$, then S will be extremized at this $t_f$ if

$$\lambda_i\ (t_f) = -\ c_i\ \ (i = 1, \ldots, n).$$

Since $\lambda_i(t_f)$ is related to $\lambda_i(t_o)$ by the $\dot{\lambda}_i$, this relation essentially specifies all $\underline{n}$ $\lambda_i(t_o)$.

Case 2 If some of the $x_1$, ..., $x_n$ are desired to have an assigned value at a fixed $t_f$, then S will be extremized and q of the $x_i$'s will be specified at this $t_f$ if q of the $\lambda_i$'s at $t_o$ are chosen so that q of the $x_i$'s at $t_f$ have specified values. Then $\lambda_i = -\ c_i$ for $i = q + 1, \ldots, n$.

Case 3 If in either of the two previous cases a fixed $t_f$ is not specified, then one additional condition is necessary to define all of the $\lambda_i$'s at $t_o$. This condition is that H at $t_f = 0$.

This formulation of the Pontryagin Principle can be applied to determining the control variables ($u_1$, ..., $u_r$) as functions of time so that the quantity S will be maximized or minimized at the right hand end point. Also, the desired boundary conditions for the original system of equations will be satisfied at the right hand end point if the $\lambda_i$'s at $t_o$ are chosen properly.

A correction to Figure 6 on page 42 of Status Report # 1 may be made here: The last element of the $\overline{c}$ vector should be -1, and the condition on $\overline{p}(t)$ should be omitted.

## C. REENTRY PROBLEM FORMULATION

Figure 1 shows the geometry of the problem with the appropriate angles and forces pictured. This geometrical presentation facilitates the mathematical formulation of the problem in a two-dimensional cartesian coordinate system with origin at the center of a non-rotating spherical earth. The y-axis of this coordinate system passes through a space fixed reference point.

The resisting force of the earth's atmosphere acting on the vehicle can be broken into two components. One component (N) acts perpendicular to the long body axis and the other (X) acts along the long body axis, and are computed by

$$X = c_X \, A \, \frac{\rho}{2} \, V^2$$

$$N = c_Z \, A \, \frac{\rho}{2} \, V^2$$

V is the magnitude of the velocity vector, $\rho$ the density of the earth's atmosphere (taken from the 1959 ARDC tables), A the frontal area of the vehicle, and $c_X$ and $c_Z$ are the aerodynamic coefficients for the vehicle. The coefficients are assumed to be functions only of the angle ($\alpha$) between the long missile axis and the velocity vector for Mach numbers larger than 2. $c_X$ and $c_Z$ are obtained from wind tunnel data and theoretical predictions of the effects of a resisting medium on the vehicle.

Expressing the axial force and the normal force in the cartesian reference system along with the components of the gravitational force in the same system produces the two components of the acceleration vector shown in Figure 1. If $x_o$, $y_o$, $\dot{x}_o$, and $\dot{y}_o$ are given, these equations can be integrated numerically to obtain x, y, $\dot{x}$, and $\dot{y}$ as functions of time if $\alpha$ as a function of time is also known. The next section explains how $\alpha$ as a function of time is to be determined.

## D. APPLICATION OF THE PONTRYAGIN PRINCIPLE

Two new variables (u and v) are introduced into the problem so that the $\ddot{x}$ and $\ddot{y}$ system of equations can be put into a form similar to the original system of equations in Section B.

$$\dot{x}_1 = \dot{x} = f_1 = u$$

$$\dot{x}_2 = \dot{y} = f_2 = v$$

$$\dot{x}_3 = \dot{u} = f_3 = \ddot{x}$$

$$\dot{x}_4 = \dot{v} = f_4 = \ddot{y}$$

In this form the system of equations has four state variables $x_1$, $x_2$, $x_3$, and $x_4$. One other extra variable is now defined.

$$x_5 = z = \int_{t_o}^{t} \left[ \left(\frac{X}{m}\right)^2 + \left(\frac{N}{m}\right)^2 \right] dt$$

Then $\dot{x}_5 = \dot{z} = f_5 = \left[ \left(\frac{X}{m}\right)^2 + \left(\frac{N}{m}\right)^2 \right].$

This allows the quantity to be extremized to be written in the form

$$S = \sum_{i=1}^{5} c_i \, x_i \, (t_f) \quad (\text{where } c_1 = c_2 = c_3 = c_4 = 0, \text{ and } c_5 = -1).$$

Therefore, $$S = -z(t_f) = -\int_{t_o}^{t_f} \left[ \left(\frac{X}{m}\right)^2 + \left(\frac{N}{m}\right)^2 \right] dt.$$

Now H for this application becomes

$$H = \lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3 + \lambda_4 f_4 + \lambda_5 f_5$$

and $$\dot{x}_i = \frac{\partial H}{\partial \lambda_i}, \qquad \dot{\lambda}_i = -\frac{\partial H}{\partial x_i} \qquad (i = 1, \ldots, 5)$$

The angle $\alpha$ in $f_1$ through $f_5$ has been taken as the control variable and must be chosen at $t_o$ and every time thereafter so that H will be a minimum with respect to $\alpha$ at these times. The minimization of H with respect to $\alpha$ at $t_o \leq t \leq t_f$ is then a requirement for $z(t_f)$ to be a minimum. Minimizing z does not necessarily insure that the peak deceleration will be a minimum, but it does imply that if the peak deceleration is large the duration will be short. This is important because the amount of deceleration that a man can stand depends both on its magnitude and its duration.

Figure 2 is an illustration of the behavior of the H function on an optimum trajectory. H itself is not shown, for reasons of better showing its variation over $\alpha$. The function H' that is shown was also used in the computations for the same reason, and is made up of only the terms in H that depend on $\alpha$. H' has its extremums at the same values of $\alpha$ that H does, but will not remain constant over the trajectory as H should do. As can be seen from the figure, three different $\alpha$'s can be chosen at every time which will produce relative minimums of H' with respect to $\alpha$ at these times. The same is true for relative maximums of H' with respect to $\alpha$. The multiple solutions which occur as the body is rotated through 360 degrees are due to the shape of the reentry vehicle assumed. Since the reentry vehicle is a three-sided body, there are three $\alpha$'s which produce relative

maxima of H with respect to $\alpha$ at $t_0 \leq t \leq t_f$. For this problem the solution nearest to a zero angle of attack ($\alpha$) is always chosen, with the other choices intended for later study.

The only other information that is needed to numerically integrate the $\dot{x}_i$'s and the $\dot{\lambda}_i$'s ($i = 1, \ldots, n$) is the n initial values for the $\lambda$'s. The statements given in Section B indicate that $\lambda_1$, $\lambda_2$, $\lambda_3$, and $\lambda_4$ should be chosen at $t_0$ so that $x_1$, $x_2$, $x_3$, and $x_4$ at $t_f$ will have the desired values. Since it is also required that S be extremized at $t_f$, $\lambda_5 = - c_5 = 1$ at $t_f$. These equations are the necessary relations which must be satisfied to minimize or maximize z at $t_f$ while also satisfying some other conditions on the $x_i$'s at $t_f$.

## E. ANALYSIS AND RESULTS

The concepts stated in the previous paragraphs have been incorporated into a computer program which determines x, y, $\dot{x}$, $\dot{y}$, and $\alpha$ as functions of time such that the integral over time of the square of the total deceleration can be either maximized or minimized at $t_f$. As input this program needs initial values for x, y, $\dot{x}$, $\dot{y}$, $\lambda_1$, $\lambda_2$, $\lambda_3$, and $\lambda_4$.

For a sample problem the cutoff conditions of a C-1 reentry test flight were used. At a 120 km altitude the velocity achieved was 8855 m/sec with a 94 degree path angle. These cutoff conditions were used to provide the x, y, $\dot{x}$, and $\dot{y}$ as input for the computer program. A systematic variation of the initial $\lambda$'s was made to observe the effects these initial $\lambda$'s have on the values of x, y, $\dot{x}$, and $\dot{y}$ at the end of the reentry phase. Figure 3 shows the type of descent caused by two different sets of initial $\lambda$'s. The shorter range trajectory has a peak deceleration of approximately 10 g's while the longer trajectory encounters only 2 g's.

## F. CONCLUSIONS

The preliminary investigation of the behavior of the descent trajectories (for the sample problem) as the initial $\lambda$'s are varied indicates that initial $\lambda$'s can be found which provide substantial variations in the range from start to finish of the reentry phase while maintaining tolerable deceleration levels. The next step in the problem will be

a mechanization of the statements presented in Section B which give conditions on the initial $\lambda$'s to produce a set of desired conditions at the lower end of the reentry phase. Also, the effect of extremizing other functions of the path will be studied. If a computer program can be developed to isolate a desired descent path automatically, then the developed procedures will become a useful tool in investigating different sets of initial conditions ($x$, $y$, $\dot{x}$, and $\dot{y}$) and different shapes of reentry vehicles. By this means then the mission and accuracy requirements needed in the development of adaptive guidance for the ascent guidance can be generated.

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# LARGE COMPUTER EXPLOITATION

By

Nolan J. Braud

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# LARGE COMPUTER EXPLOITATION

By

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## SECTION I. INTRODUCTION

The first status report lists three major areas of scientific disciplines which are being investigated to advance the state-of-the-art in achieving numerical results of problems in support of Space Flight and Guidance Theory. These areas are a function differential generator, the development of a statistical model, and investigations of multivariant functional models. The purpose of this report is to briefly indicate the status of development in these three areas since the publication of the first report.

## SECTION II. FUNCTION DIFFERENTIAL GENERATOR

The function differential generator is a computer program that will differentiate a certain class of algebraic and transcendental expressions automatically, where the allowable class of expressions must be closed under differentiation. At the time of the first status report, the University of North Carolina had developed a working model which had been successfully employed in generating the coefficients for the Taylor's Series expressions of the simplified flat-earth calculus of variations problem. Only first order derivatives were evaluated; however, since then extension of size limitations and other characteristics were added so that higher order derivatives could be obtained.

Some of the features which are being incorporated into the program are:

(1) An increase in the limit of length of the input string to more than twice its original size.

(2) An increase of the limit of length of the string which may be generated to 4200 alpha-numeric characters.

(3) The elimination of duplicate values in the matrix which is to be operated on.

(4) The inclusion of a symbolic differential operator so that chain differentiation can be accomplished.

(5) The ability to evaluate derivatives and partial derivatives.

Upon completion of these modifications the program will be employed to generate the Taylor's Series of second order. Then convergence and error properties will be investigated.

## SECTION III. STATISTICAL MODEL DEVELOPMENT

The disciplines involved in statistical model development are fairly well defined. The procedures are straightforward and very little changes in operation were introduced since the last report. The single major revision was the incorporation of the "Three Dimensional Optimum Trajectory Program" by Auburn University (Reference 1). The oblateness of the earth was later included in these equations and three dimensional (position wise) statistical models are now being used. This results in the nature of the steering and cutoff functions in the path adaptive guidance mode being

$$\left.\begin{matrix} \chi_o \\ T_f \end{matrix}\right| \Bigg\} = \begin{matrix} \chi_o \\ T_f \end{matrix} \left[ x_o,\ y_o,\ z_o,\ \dot{x}_o,\ \dot{y}_o,\ \dot{z}_o,\ (F/m)_o,\ t_o \right]$$

where the "o" subscript refers to instantaneous conditions. It may be noted that the $(\dot{m}/m)_o$ term as shown in the first status report has been dropped from consideration for the time being, because it cannot presently be determined accurately enough during the flight of a vehicle.

## SECTION IV. MULTIVARIANT FUNCTIONAL MODELS

The comparison of various multivariant function approximating procedures is being continued with major emphasis being devoted to least squares and linear programming techniques. The least squares methods are still providing the majority of useful results but the linear programming investigations have not been carried far enough to come to any conclusive decision as to a preference.

Chrysler Corporation and Northeast Louisiana State College are devoting their efforts to investigations in the area of least squares techniques while the University of North Carolina is investigating linear programming. The efforts of Northeast have resulted in the development of sufficiency conditions for the existence of multivariant least squares approximating functions, whereas the use of orthonormal polynomials is being

investigated by Chrysler as a means of writing guidance equations. By the use of orthonormal polynomials, one gains control of errors introduced by computation procedures. This is brought out by the fact that when normal equations of excessive length are used in the determination of least squares coefficients, ill-conditioned or singular matrices are often encountered. This doesn't happen when orthonormal polynomials are used. The theory behind this technique of determining least squares polynomials and an indication of means to select the "best" set of tabulated data for use in the numerical approximating procedure is contained in Reference 2.

North Carolina is endeavoring to increase the speed of convergence of the revised simplex method to the solution of the dual problem which appears in the linear programming procedure. The linear programming procedure requires operation on an expression which is derived from an inequality condition. The fundamental problem as normally stated is referred to as the <u>primal</u> problem; however, the problem can be reformulated when the inequality is reversed, in which case it is referred to as the <u>dual</u>. In doing this, advantage is taken of the duality theorem which states in essence that if the primal problem has a solution then the dual problem also has a solution.

Normally this pair of problems has constraints which for one problem equals the number of data points and for the other problem equals the number of unknown coefficients in the approximating function. The number of constraints directly influences the speed of convergence in each problem, and since the number of data points is much greater than the number of unknown coefficients, the latter may be the most economical problem to solve. A formulation of this nature has been made by the University of North Carolina and will be evaluated by MSFC.

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# APPLICATIONS OF SPACE FLIGHT AND GUIDANCE THEORY

By

D. H. Schmieder

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# APPLICATIONS OF SPACE FLIGHT AND GUIDANCE THEORY

By

D. H. Schmieder

## SECTION I. INTRODUCTION

The scientific discipline development with which the preceding chapters have been chiefly concerned has two attributes which may be mentioned here. First, the studies being made in that area are slanted toward gaining an understanding of the nature of the problems with which we are faced, isolating and resolving the difficulties encountered when known theory and techniques are applied to these problems, and attempting to extend theory and techniques where most desirable. Second, the end result that is desired is the ability to carry out the applications to the Saturn program that are required of this branch more economically and rigorously.

The significant results of such development work clearly cannot be scheduled to coincide with the desires of an applications program, but the applications program can indicate where advances would be most beneficial, and it makes some sense to distribute the scientific discipline development effort accordingly. The applications are then carried out with the best theory and techniques available at the time the applications are required.

The following paragraphs describe some of the applications that have been initiated since Status Report #1 in Future Projects Branch. They should serve to illustrate where the scientific discipline development is or will be used, and part of what is needed.

## SECTION II. DISCUSSIONS

### A. LUNAR ORBITAL RENDEZVOUS STUDIES

An application of some extent was that of determining flight profiles and launch windows for flights to the moon and back, in connection with a Lunar Orbital Rendezvous feasibility study. This application was not completed by any means in the technical sense, but results were obtained on some of the more

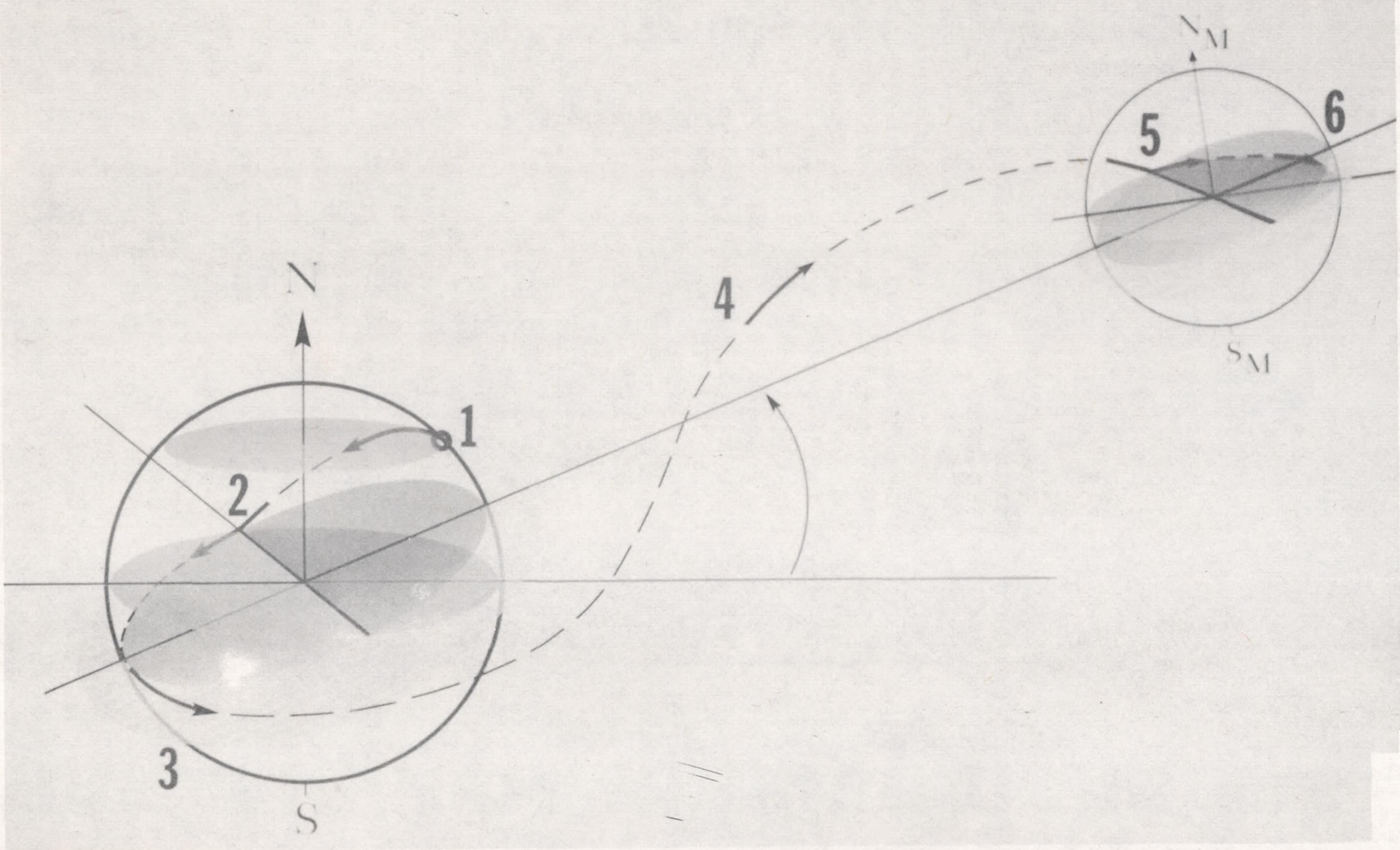
important points and presented to Dr. Shea of Office of Manned Space Flight on June 7, 1962. Most of the results are being published as a number of individual studies (References 1 to 5).

The general problem may be described with the help of Figure 1. A space vehicle of the Saturn class is to begin its mission at point 1 with a launch from the latitude of Cape Canaveral. At point 2 a circular orbit would be obtained, and at point 3 an injection would be made into a lunar trajectory. A number of midcourse maneuvers would be possible during the translunar flight (point 4), followed by a braking phase into lunar satellite orbit at 5. At point 6 part of the spacecraft would brake further into a landing trajectory, and would later launch back off the surface of the moon to rendezvous with the part that stayed in orbit. The spacecraft would then inject into a return trajectory, which after possible midcourse maneuvers would reenter the earth's atmosphere in such a way as to be able to safely decelerate by means of aerodynamic drag and land back on earth. Many practical considerations and physical effects enter in, such as a safe and instrumented launch azimuth and return approach; accelerations and aerodynamic heating on ascent and reentry; radiation belt and solar radiation in transit flight; suitability of lunar landing sites and their effect on choice of lunar orbit; relative motion, spin rates, and spin orientation of earth and moon as a function of date; communication and abort possibilities at most points; and performance and guidance considerations.

A reasonable approximation to the optimum flight profile for this problem should not be too difficult to determine if analytical solutions to all phases of the flight were available. However, such solutions are not presently available and represent some of the aims of the scientific discipline development. Therefore, numerical approaches must be applied, and due to the extreme complexity of the problem, various phases must be treated independently with assumptions that are judged to be reasonable made on the remaining phases. The midcourse maneuver and the lunar landing phases were not studied by this branch.

A study of the return leg from lunar satellite orbit to suitable reentry conditions is reported on in Reference 1. The injection conditions at the moon for landing at either San Antonio, Texas, or Woomera, Australia was investigated for various declinations of the moon. The survey was made on a deck obtained from the Jet Propulsion Laboratory that, within its computational accuracy, is as representative of the true celestial situation as any known today.

# POSSIBLE BURNING-PHASES FOR PLANE-CHANGES (SCHEMATICS)



Another study that was run on the same deck investigated the outgoing leg from earth to periselenum at various definite dates when such a lunar mission would be likely to be flown. The twisting effect found in these realistic non-coplanar lunar trajectories was employed to achieve the smallest inclination of the lunar arrival conic possible without any powered "plane changing." Also, some of the principal effects on the trajectories due to properties of the celestial model not included in simplified decks were measured. These studies are reported in Reference 3.

To get some picture of the effect of injection parameters on periselenum conditions on a somewhat broader scale, a survey of such trajectories was run on a simplified deck based on the restricted three body model (earth and moon traveling in circles about the barycenter, vehicle's mass not affecting their motion). The relationships between injection parameters and periselenum conditions that were found numerically are shown graphically in Reference 2. These results are given in a moon-earth-plane (MEP) coordinate system. A transformation of these results to geo-and selenographic coordinates, with flight planes being measured with respect to lunar and earth's equators, has been computed for all expected orientations of the two equatorial planes, and will be published. Also, a means of determining the actual relationship between these coordinate systems on any particular date has been published preliminarily in Aeroballistics Division (Reference 4).

The fact that only numerical representations of the earth-moon transits are presently available makes more difficult the optimization of the powered injection at earth, and brake and injection at moon, since part of the end conditions for these phases are not expressible in equation form. Therefore, the first studies involving these powered phases have been made assuming only a certain energy level and flight plane to be required at the terminals of the powered phases. It is felt that the corresponding results were a fair approximation, especially concerning the expenses of plane changes. Some of the results were given in the presentation to the Office of Manned Space Flight, NASA Headquarters. More complete results will be published individually in the future.

Two other possibilities for plane changes have been studied by Future Projects Branch: during ascent from lift-off to circular orbit of earth, and during a transfer from one circular orbit to another of equal or different radius. The orbit to orbit work has not been completed. Most of the results on plane changing during ascent phase are being expressed in terms of "launch windows."

It is probable that the most economical flight profile for the lunar orbital mission, independent of firing date and instant, would involve no powered plane changing. However, for various reasons it is necessary to have an interval of time on a number of dates when the flight can begin, making plane changes necessary. These possible firing intervals, called "launch windows," depend on the flight profile that is planned. For example, it may be better due to engineering constraints such as tracking to launch on a given azimuth, even though the launch window could be enlarged by having a variable launch azimuth.

Each time there is a non-powered phase in the flight profile, the launch window question arises for the ignition of the next powered phase ("ignition window"). Some information for most such windows occurring in the LOR mission was given in the June 7 presentation. A more detailed study for the ascent phase was completed later, based on optimum three-dimensional trajectories injecting into a space fixed circular orbit, and the results were presented at the ARS Meeting in Los Angeles, California, November 13-18, 1962 (Reference 5). Results for the injection into the transit from moon to earth are included in Reference 1.

B. LUNAR LOGISTICS SYSTEMS STUDIES

An applications problem which is very much related to the LOR problem is that of the Lunar Logistics Systems (LLS). The mission is to soft-land cargo at a given location on the moon with a C-5 or C-1B type of vehicle, either directly, or by first achieving a lunar satellite orbit.

Studies have been made for the earth-moon-transit similar to those made for LOR, but more extensively. Relationships between injection and arrival conditions have been computed and compared on the simplified deck (MEP Coordinates) over a wide class of lunar arrival conditions. These conditions range continuously over and beyond the face of the moon, from retrograde approaches that are suitable for lunar satellite orbit injections, thru lunar impact approaches, to the probably impractical but instructive direct approaches that are also available for injecting into lunar satellite orbit. Effects of using the more exact models of the earth-moon system and interpretations in geo- and selenographic coordinate systems have also been computed. The results are being given in special LLS presentations and will appear later in more detail in the Lunar Flight Study Series of which Volumes 1 to 3 are published, (References 1 to 3). Some representative trajectories for this problem computed on the JPL deck have been published (Reference 6).

The work done for the LOR concerning the propelled flight phases of ascent to circular orbit, injection into transit, and braking into lunar satellite orbit is equally applicable to the LLS problem. The work was extended to compare C-5 and C-1B values for velocity budget and launch windows, and to give results on the way in which the lunar landing phase extends the area of the moon achievable by impact type of approaches, from vertical to horizontal. Plane changes during the braking phase were also studied, both as a possible means of reaching given landing sites, and as a necessary means of approaching given landing sites from a given direction on any date. The results of these studies will be given in a manner similar to that of the translunar work.

## C. ADAPTIVE GUIDANCE APPLICATIONS

An example of the techniques used in arriving at guidance functions for particular missions was given in the applications section of Status Report #1 (Reference 7). Since the time of that report, a determination of the effect of segmenting the polynomial approximation to an upper stage steering function for injection into circular orbit has been made. The time point separating the two segments was varied, and the best approximations obtainable by least squares methods were compared for various numbers of terms in the polynomial. Of course, it is impossible to consider numerically all possibilities for a given number of terms, but one approach to making this determination was investigated. A description of these studies and their results will be published in report form soon.

The best steering functions available by these and similar techniques were computed as a preliminary working set for the SA-7 application. SA-7 has an orbital injection mission and will be the first time the adaptive guidance equations are actively flown. SA-5 has the same mission and will be guided by other means, with all adaptive guidance functions being computed but not followed. This set of guidance functions is presently being modified to meet certain hardware requirements.

## D. COASTING PERIODS DURING ASCENT PHASE

One possible means of increasing flexibility in the attainment of strongly range dependent missions, such as rendezvous, would be to include a coasting period of variable length at some variable time during ascent. This problem is being treated by the Boeing personnel attached to Future Projects Branch under a general C-5 contract. The mission is taken to be a two-dimensional rendezvous with another vehicle that is already traveling

in a low circular orbit about the earth. The coast period is permitted to occur at any time (restart capability assumed) after the end of first stage. The coasting phase is constrained to a minimum altitude of 100 km. The optimum trajectory and coasting parameters are isolated for various assumed values of the orbit altitude and first stage performance parameters. The corresponding burnout weights provide information as to launch windows. The results will be published when suitable portions have been completed.

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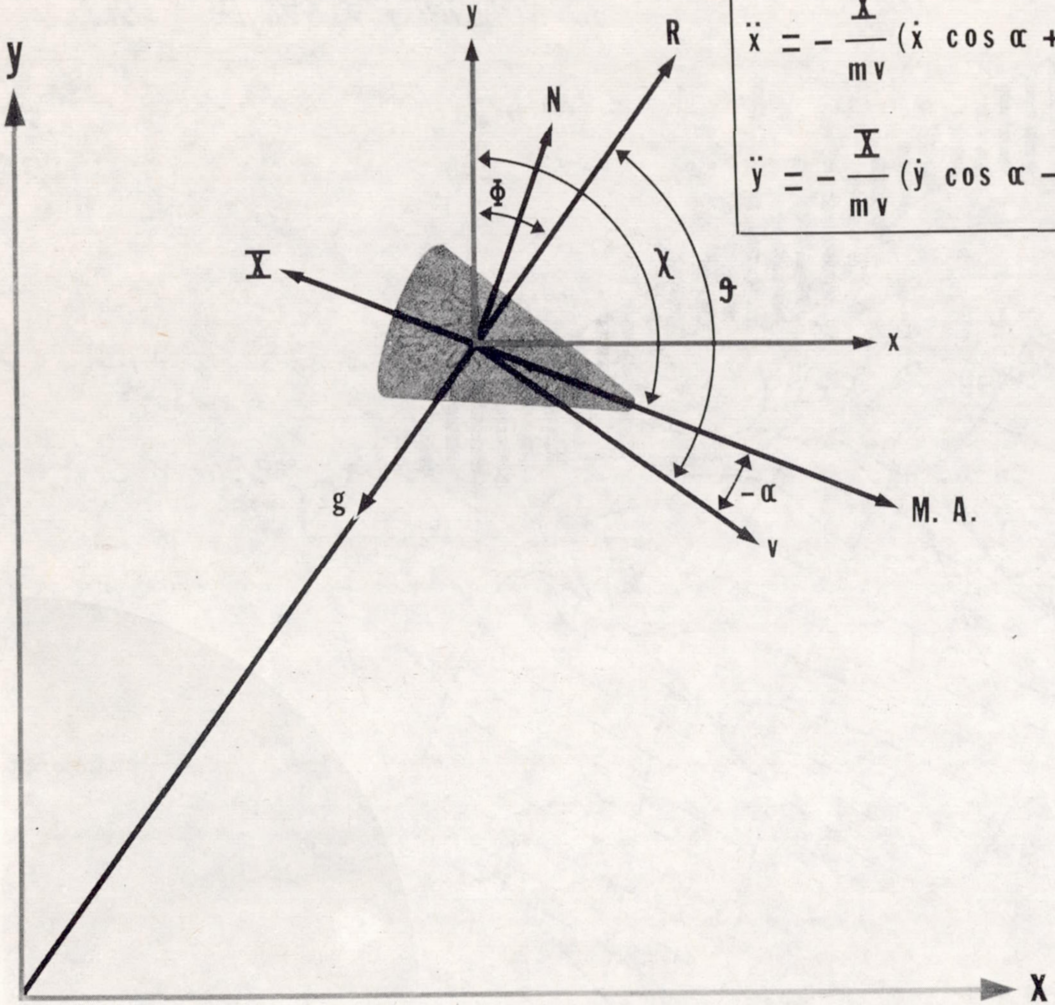


**EQUATIONS OF MOTION**

$$\ddot{x} = -\frac{\mathbb{X}}{mv}(\dot{x}\cos\alpha + \dot{y}\sin\alpha) - \frac{N}{mv}(\dot{y}\cos\alpha - \dot{x}\sin\alpha) - \frac{K^2 x}{R^3}$$

$$\ddot{y} = -\frac{\mathbb{X}}{mv}(\dot{y}\cos\alpha - \dot{x}\sin\alpha) + \frac{N}{mv}(\dot{x}\cos\alpha + \dot{y}\sin\alpha) - \frac{K^2 y}{R^3}$$

**FUNCTION TO BE EXTREMIZED**

$$z = \int_{t_0}^{t_f} \left[\left(\frac{\mathbb{X}}{m}\right)^2 + \left(\frac{N}{m}\right)^2\right] dt$$

*FIG. 1. REENTRY PROBLEM FORMULATION*

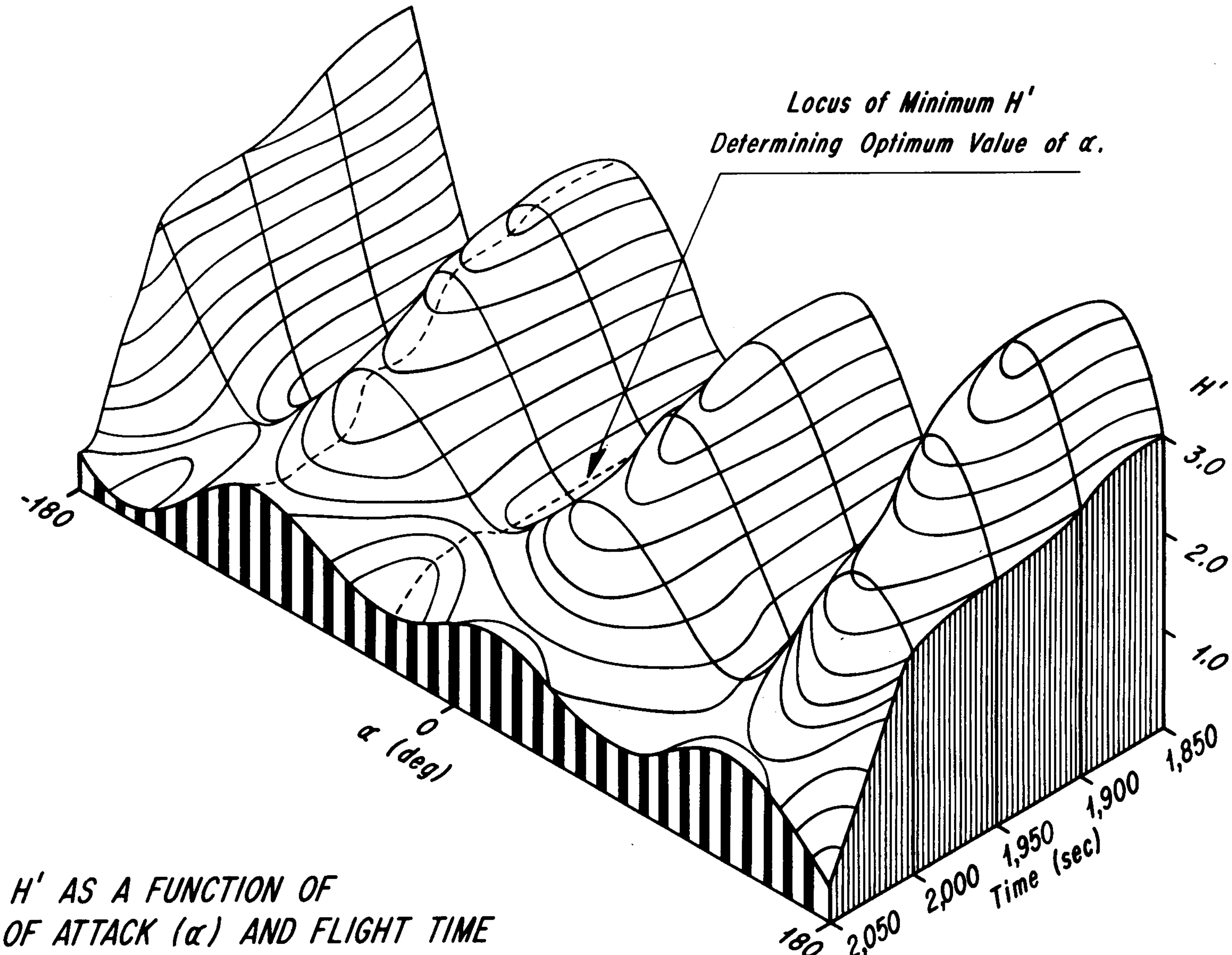


FIG. 2. H' AS A FUNCTION OF
ANGLE OF ATTACK (α) AND FLIGHT TIME
OVER A PORTION OF A TYPICAL REENTRY TRAJECTORY

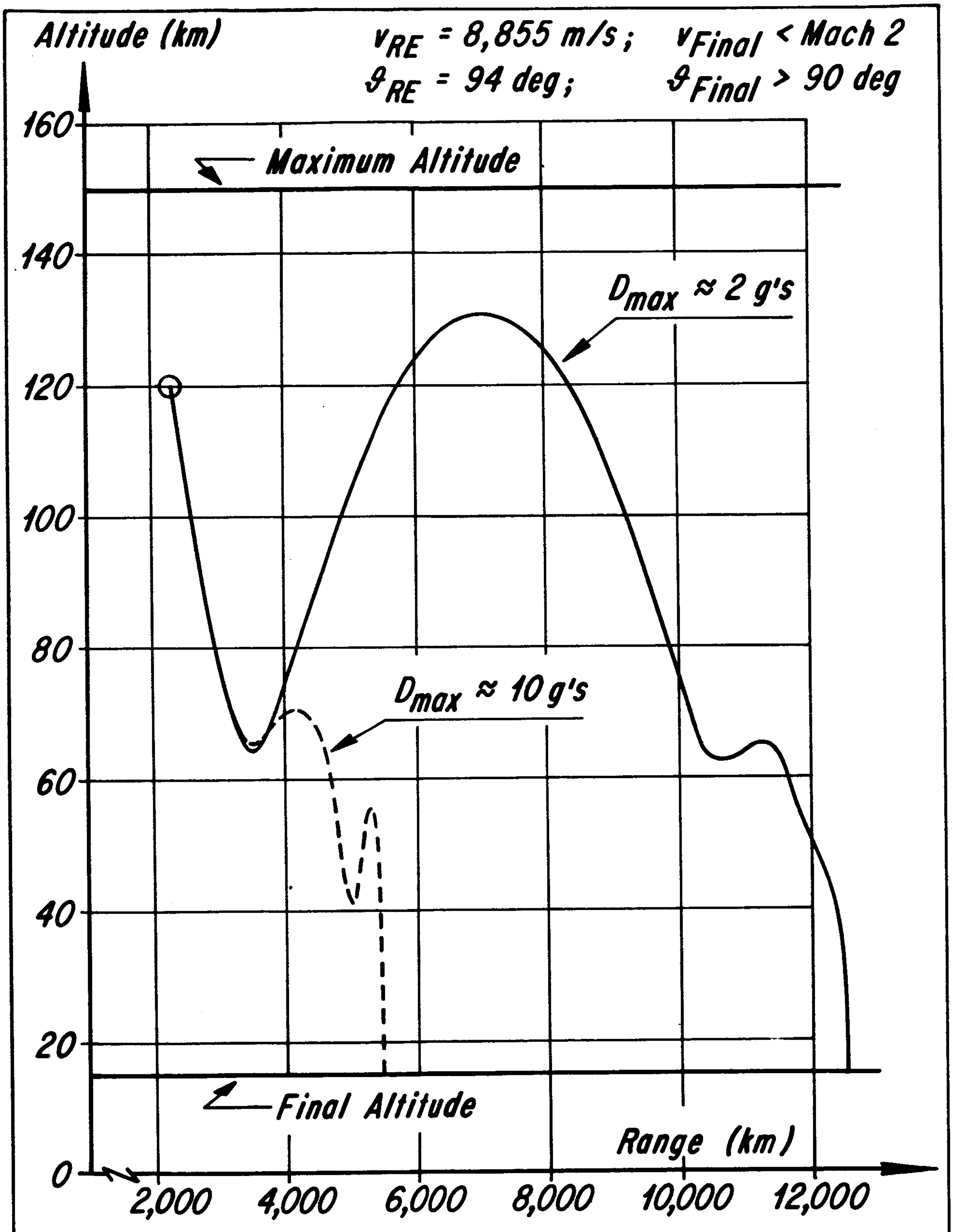


FIG. 3. TYPICAL TRAJECTORIES

APPROVAL

STATUS REPORT #2
on
THEORY OF SPACE FLIGHT AND ADAPTIVE GUIDANCE

Coordinated By
W. E. Miner

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be Unclassified.

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